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Tax Compliance and Rank Dependent Expected Utility

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Abstract

Formulating the classic Allingham and Sandmo [1972] tax compliance problem under Rank Dependent Expected Utility (RDEU) provides a simple explanation for the "excess" level of full compliance observed in empirical studies, which standard Expected Utility (EU) theory is unable to explain. RDEU provides a compelling answer to this puzzle, without the need for the moral sentiments or stigma arguments that have recently been advanced in the literature. Formally, we show that the threshold audit probability or penalty rate at which full compliance becomes optimal for the decisionmaker are significantly lower under RDEU axiomatics than in the EU case, and that the optimal level of underreporting is lower under RDEU. Numerical simulations using various parameterizations of the probability weighting function illustrate the large quantitative differences between the two models, while a simulation of underreporting rates in the US over the past 50 years shows how RDEU can go some way towards explaining the tax-compliance puzzle.

Key words: rank-dependent expected utility, tax-compliance

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1. Introduction

Since the seminal article by Allingham and Sandmo [1972], most authors dealing with the issue of tax compliance have had great difficulty in making their theoretical models square with empirical or experimental results. The most glaring example is the tendency of many individuals to engage in *no* tax evasion at all, whereas the Allingham and Sandmo [1972] model predicts, when the expected rate of return to each dollar of taxes evaded is strictly positive (which is the case in the US, for example), that *all* risk-averse taxpayers will underreport their income. Despite the "over-compliance" of individuals, at least with respect to what is predicted by an expected utility (EU) model, tax-evasion is an enormous problem: in the US alone, tax-evasion is estimated by the Internal Revenue Service (IRS) to amount to $300 billion per year.

A first common reaction to the over-compliance puzzle has been to "improve" upon the initial model. Yitzhaki's [1974] contribution was to assume that the penalty for non-compliance is proportional to the amount of taxes evaded, while Pencavel [1979] endogenized income by jointly considering labor supply along with tax compliance. Koskela [1983] considered the nature of penalty schemes (charged on undeclared income or on the amount of tax evaded). Pestieau and Possen [1991], for their part, included the choice of activity by the consumer, where each sector varies in the opportunities available for evasion. Engel and Hines [1999] focused upon the repeated nature of the problem and explored its dynamics. Finally Graetz, Reinganum, and Wilde [1986], and Beck and Jung [1989] introduced strategic concerns: the former used game-theoretic tools to model the interaction between the taxpayer and the tax authorities; the latter endogenized the audit probability within a principal-agent framework, in which the audit probability becomes a function of the amount of income declared.

While these developments have been both interesting and important, most observers agree that they still do not allow one to reconcile theory with observed empirics (see e.g. the survey by Andreoni, Erard, and Feinstein [1998]). As a result, a number of authors have introduced new constraints, derived from psychological arguments, in an effort to explain the "excessive" level of observed compliance. Spicer and Lundstedt [1976], for example, considered the degree of satisfaction felt by the taxpayer with respect to his government. Erard and Feinstein [1994] included notions of guilt and shame in the taxpayer's objective function. While the heuristic appeal of these arguments is undeniable, they remain, however, difficult to justify on purely economic grounds.

A second approach has been to raise doubts concerning the expected utility framework initially adopted by Allingham and Sandmo [1972]. Slemrod [1992], for example, summarizes a large corpus of empirical and experimental literature that finds a subjective probability of audit that is significantly different from (and larger than) the observed objective probability. Our paper picks up on this idea as its point of departure, and takes aim at the fundamental building-block of the Allingham and Sandmo [1972] approach: the EU model of von Neumann and Morgenstern. As such, we formalize the Allingham and Sandmo [1972] problem under alternative axiomatics, using the Rank-Dependent Expected Utility (RDEU) model.

The RDEU approach was initially developed by Quiggin [1982] in order to address a number of important weaknesses that had become apparent in the EU approach. Under RDEU, the linearity in probabilities of the EU model is replaced by a probability weighting or perception function (see Chateauneuf, Cohen, and Meilijson [2005]) which assigns weights to the probabilities of the different states of nature, where the weights are themselves functions of the rank of the given state of nature, in terms of the level of satisfaction that the individual derives.[1] Bernasconi [1998] analyzes tax compliance using the notion of first-order risk-aversion, introduced into the literature by Segal and Spivak [1990]. In a two states of nature example (which corresponds exactly to the Allingham and Sandmo tax compliance problem) the particularity of Segal and Spivak's approach is that an individual's indifference curves possess a kink along the 45 degree line (which corresponds to perfect insurance). Formally, individual preferences admit points of non-differentiability, where risk-aversion is of order one. This property arises naturally under RDEU axiomatics. Indeed, Bernasconi [1998] illustrates his results using a numerical simulation based on a RDEU model in which the parameterization of the probability perception function is borrowed from the empirical work of Camerer and Ho [1994]. Our paper can thus be seen as a natural extension to Bernasconi's work, in which RDEU axiomatics are posed both explicitly and right from the start, and in which a broader set of parameterizations of the probability weighting function is tested in numerical simulations.

The paper is organized as follows. In part 2 we introduce the Allingham and Sandmo problem in terms of RDEU axiomatics, and present our main theoretical result which shows that full compliance is "easier" to obtain under RDEU than under EU. Formally speaking, this is expressed by the threshold audit probability at which full compliance becomes optimal for the consumer being lower under RDEU than in the EU case. It follows, under certain conditions, that the level of underreporting will be lower under RDEU than under EU. In part 3, we perform numerical simulations using the parameterizations of the probability weighting function proposed by Camerer and Ho [1994], Tversky and Fox [1995] and Prelec [1998], and examine: (i) the minimal audit probability and penalty rate needed to ensure full compliance, and (ii) the compliance rate (this is done for various specifications of the utility function as well). For realistic parameter values drawn from US evidence, the underreporting rates predicted by our theoretical models are then compared with the observed levels of underreporting, over the past 50 years. These simulations show that RDEU, and the pessimistic attitudes it can account for, can provide part of the explanation for the tax-compliance puzzle.

2. Allingham and Sandmo [1972] under RDEU

2.1. *The model*

In the Yitzhaki's [1974] version of the standard Allingham and Sandmo [1972] problem, the penalty faced by the taxpayer in the case of an audit is proportional to the amount of tax avoided, when the taxpayer engages in a positive amount of avoidance. The lottery faced by the taxpayer is given by $P(z) = (p, y - \theta tz; 1 - p, y + tz)$, where p is the probability of being audited, y is the taxpayer's after tax income, t is the tax rate, θ is the penalty rate if fraud is detected, and z is the amount of underreporting by the taxpayer.[2]

The taxpayer's problem under RDEU axiomatics is then given by

$$\max_{z\geq 0} RDEU[P(z)] = \max_{z\geq 0} \varphi(1-p)u(y+tz) + (1-\varphi(1-p))u(y-\theta tz), \tag{1}$$

where $u : \mathbb{R} \longrightarrow \mathbb{R}$, defined up to a monotone increasing transformation, plays the role of a utility function under certainty, and $\varphi : [0, 1] \longrightarrow [0, 1]$, which satisfies the restrictions $\varphi(0) = 0$ and $\varphi(1) = 1$, is unique and plays the role of a probability transformation function; u and φ are both continuous and increasing.[3] The solution to the taxpayer's optimization problem is characterized by the necessary First Order Condition (FOC):

$$t[\varphi(1-p)u'(y+tz^*_{RDEU}) - \theta[1-\varphi(1-p)]u'(y-\theta tz^*_{RDEU})] + \lambda = 0, \tag{2}$$

where λ is the Lagrange multiplier associated with the constraint $z \geq 0$.[4] Two cases will arise because of the complementary slackness condition from Kuhn-Tucker: $\lambda z^*_{RDEU} = 0$.

First, we may have $z^*_{RDEU} = 0$ and $\lambda > 0$. Rearranging (2) implies that:

$$-\lambda = t\theta[1-\varphi(1-p)]u'(y)\left[\frac{\varphi(1-p)}{\theta[1-\varphi(1-p)]} - 1\right] < 0,$$

which can only be true for $(1+\theta)\varphi(1-p) - \theta < 0$, and becomes impossible when $(1+\theta)\varphi(1-p) - \theta > 0$. Second, we may have $z^*_{RDEU} > 0$ and $\lambda = 0$, and (2) can be rewritten as:

$$\frac{\varphi(1-p)}{\theta[1-\varphi(1-p)]} = \frac{u'(y-\theta t z^*_{RDEU})}{u'(y+t z^*_{RDEU})} \equiv f(z^*_{RDEU}; y, t, \theta). \tag{3}$$

By inspection of (3) it is immediate that $f_{z^*}(z^*_{RDEU}; .) > 0$.[5] Moreover, it is equally clear that $f(0; .) = 1$. Therefore (3) cannot hold when $(1+\theta)\varphi(1-p) - \theta < 0$ and can only obtain when $(1+\theta)\varphi(1-p) - \theta \geq 0$. Consider the partial inverse of f with respect to z^*_{RDEU}, which we shall denote by ψ, where $\psi(f(z^*_{RDEU}; y, t, \theta); y, t, \theta) = z^*_{RDEU}$. It follows that $\psi(f(0; .); .) = \psi(1; .) = 0$. Since f is increasing, so is its inverse: $\psi_f(f; .) > 0$. Since $\varphi(.)$ is a strictly increasing function from [0, 1] to [0, 1], its inverse $\varphi^{-1}(.)$ is so as well and the condition $\varphi(1-p) - \theta[1-\varphi(1-p)] \geq 0$ can be rewritten as:

$$p \leq 1 - \varphi^{-1}\left(\frac{\theta}{1+\theta}\right) = \underline{p}^*_{RDEU}(\theta).$$

We then have the following Proposition:

Proposition 1: *The taxpayer's optimal compliance behavior is given by*:

$$z^*_{RDEU}(p; y, t, \theta) = \begin{cases} \psi\left(\dfrac{\varphi(1-p)}{\theta[1-\varphi(1-p)]}; y, t, \theta\right), & p < \underline{p}^*_{RDEU}(\theta) \\ 0, & \textit{otherwise} \end{cases} \tag{4}$$

If we pose $\varphi(p) = p$, $\forall\, p \in [0, 1]$, then we are back to EU axiomatics, and one obtains:

$$z^*_{EU}(p; y, t, \theta) = \begin{cases} \psi\left(\dfrac{1-p}{\theta p}; y, t, \theta\right), & 1 - p - \theta p \geq 0 \\ 0, & \textit{otherwise} \end{cases} \tag{5}$$

This result is a standard one in the tax-compliance literature (see e.g. Andreoni, Erard, and Feinstein [1998]).

Experimental studies show that the probability weighting function ($\varphi(.)$) is inverse S-shaped (first concave, then convex), overweighting low probabilities and underweighting high probabilities [Heath and Tversky, 1991; Abdellaoui, 2000]. The probability weighting function therefore satisfies the condition that $\exists\ \hat{p} \in [0, 1]$, such that $\varphi(\hat{p}) = \hat{p}$, with $\varphi(p) > p, \forall p < \hat{p}$ and $\varphi(p) < p, \forall p > \hat{p}$. For Prelec [1998], "the overweighting of small probabilities, below the fixed point [$\hat{p}$], enhances the attraction of small-p gains (lottery tickets) and the aversion to small-p losses [audit], while the underweighting of larger probabilities above the fixed point, diminishes the attraction of larger-p gains [underdeclaration without auditing] and the aversion to larger-p losses." Prelec [1998] also establishes that $\hat{p}$ lies between 0.2 and 0.4. Decidue and Wakker [2001] specify that: "Descriptively, a pessimistic attitude can result from irrational belief that unfavorable events tend to happen more often, leading to an unrealistic overweighting of unfavorable likehoods (Murphy's law)."

In what follows, we assume that the true audit probability q and the penalty rate faced by the taxpayer satisfy:

Condition 1. (i) $\varphi(1-q) < 1-q$; (ii) $\varphi(\frac{\theta}{1+\theta}) < \frac{\theta}{1+\theta}$.

When $\varphi(p)$ is inverse S-shaped, Condition 1(i) boils down to assuming that the true audit probability q is strictly smaller than the fixed point $\hat{p}$, an hypothesis that would appear reasonable in light of the available estimates of the audit probability ($q \in [0.01, 0.05]$, Andreoni, Erard, and Feinstein [1998]) and of the fixed point ($\hat{p} \in [0.2, 0.4]$).[6] Condition 1(ii) implies that the penalty rate must be sufficiently "high". For an inverse S-shaped $\varphi(.)$, it is equivalent to $\theta > \frac{\hat{p}}{1-\hat{p}}$. Again, this condition is reasonable in "real world" terms as $\hat{p} = 0.2$, for example, implies $\theta > 0.25$ (the minimum rate in the US is 0.20). We are now in a position to compare $z^*_{EU}(q; y, t, \theta)$ and $z^*_{RDEU}(q; y, t, \theta)$:

Proposition 2: *Consider $z^*_{RDEU}(q; y, t, \theta)$ and $z^*_{EU}(q; y, t, \theta)$ as defined in Eqs.* (4) *and* (5). *Then, when Condition* 1(ii) is satisfied, $\underline{p}^*_{RDEU}(\theta) < \underline{p}^*_{EU}(\theta)$. Moreover, when Condition 1(i) is satisfied:

(i) for $q < \underline{p}^*_{RDEU}(\theta)$, $z^*_{EU}(q; .) > z^*_{RDEU}(q; .) > 0$;
(ii) for $\underline{p}^*_{RDEU}(\theta) \leq q < \underline{p}^*_{EU}(\theta)$, $z^*_{EU}(q; .) > z^*_{RDEU}(q; .) = 0$;
(iii) for $\underline{p}^*_{EU}(\theta) \leq q$, $z^*_{EU}(q; .) = z^*_{RDEU}(q; .) = 0$.

Proof. Condition 1(ii) can be rewritten as $\underline{p}^*_{EU}(\theta) = \frac{1}{1+\theta} < 1 - \varphi^{-1}(\frac{\theta}{1+\theta}) = \underline{p}^*_{RDEU}(\theta)$. Rewrite Condition 1(i) as $1 - q - \varphi(1-q) > 0$. Adding $q\varphi(1-q)$ to both sides yields $(1-q)[1-\varphi(1-q)] > q\varphi(1-q)$. Rearranging this inequality and dividing both sides by θ yields: $\frac{1-q}{\theta q} > \frac{\varphi(1-q)}{\theta[1-\varphi(1-q)]}$. Since $\psi_f(f; .) > 0$, it follows that $\psi(\frac{1-q}{\theta q}; .) > \psi(\frac{\varphi(1-q)}{\theta[1-\varphi(1-q)]}; .)$, which implies, by Proposition 1, that $z^*_{EU}(q; .) \geq z^*_{RDEU}(q; .)$. The rest of Proposition 2 is immediate. □

In the original contribution by Allingham and Sandmo [1972], the main factor limiting tax avoidance is the consumer's risk-aversion; RDEU axiomatics allow one to add pessimism to the picture, in the sense of the consumer's overweighting of lower-ranked outcomes (in this case, being audited). The pessimism of individuals leads them to a greater degree of compliance than in the EU case.[7]

Proposition 2 is illustrated in figure 1, using the single parameter probability weighting function proposed by Kahneman and Tversky [1992].[8] Full compliance obtains when the

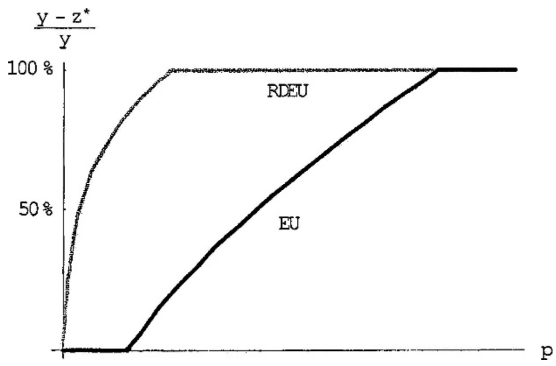

Figure 1. Compliance rate as a function of the audit probability.

compliance rate curves reach the 100% level. As should be obvious from figure 1, this occurs for much lower values of p under RDEU than in the EU case.

3. Simulation results

In the numerical simulations that follow, we shall consider the following parameterizations of the probability weighting function: Camerer and Ho (1994), which is equivalent to Kahneman and Tversky [1992] with $\gamma = 0.56$, as used by Lattimore, Baker, and Witte [1992], Gonzalez and Wu [1999]; Tversky and Fox (1995): $\frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$, $\delta = 0.77, \gamma = 0.69$; Prelec (1998): $\exp[-\delta(-\ln[p])^{\alpha}]$, as specified by Bleichrodt and Pinto [2000]: $\delta = 1.08$ and $\alpha = 0.53$.[9] Bernasconi [1998] uses the Camerer and Ho [1994] probability weighting function in his work on tax avoidance. It is clear at this stage that experimental work, in the specific context of tax-avoidance, would be extremely useful in setting the parameter values correctly in the simulations that follow.[10] Given the lack of such experimental evidence, our results should be taken with a grain of salt, although they are likely to be representative of the broad differences between the EU and RDEU cases.

3.1. Threshold audit probabilities and penalty rates

Proposition 2 establishes a relationship between the audit probability and the penalty rate that ensures full compliance, expressed in terms of the threshold audit probabilities: $\underline{p}^*_{EU}(\theta) = \frac{1}{1+\theta}$ and $\underline{p}^*_{RDEU}(\theta) = 1 - \varphi^{-1}(\frac{\theta}{1+\theta})$. Now note that $\frac{\partial(\underline{p}^*_{EU}(\theta)-\underline{p}^*_{RDEU}(\theta))}{\partial\theta} = -\frac{1}{(1+\theta)^2}[1 + \varphi^{-1'}(\frac{\theta}{1+\theta})] < 0$. The difference between $\underline{p}^*_{EU}(\theta)$ and $\underline{p}^*_{RDEU}(\theta)$ is thus a decreasing function of the penalty rate θ. This statement can be formulated in an alternative manner by seeking to determine the minimal penalty rate that entails full compliance, for a given value of the true probability of audit q. Formally-speaking, these "limit" penalty rates can be expressed as $\theta^*_{EU} = \frac{1-q}{q}$ and $\theta^*_{RDEU} = \frac{\varphi(1-q)}{1-\varphi(1-q)}$. If we assume that the true audit probability satisfies Condition 1(i), it is immediate that $\theta^*_{EU} > \theta^*_{RDEU}$. It follows that when the penalty rate

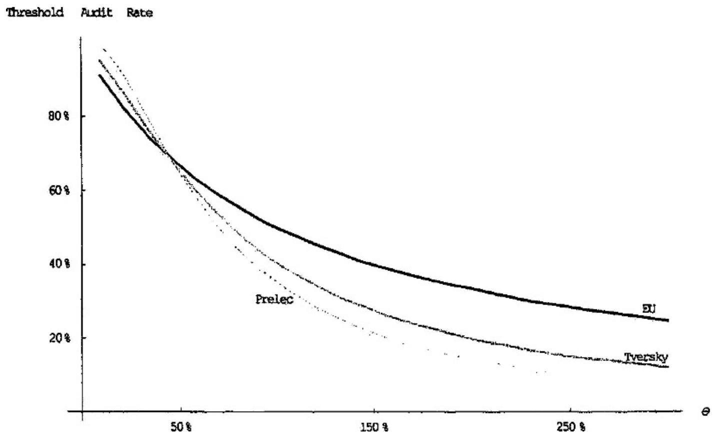

Figure 2. Threshold audit probabilities ($\underline{p}^*_{RDEU}(\theta)$ and $\underline{p}^*_{EU}(\theta)$) that ensure full compliance, as function of the penalty rate θ.

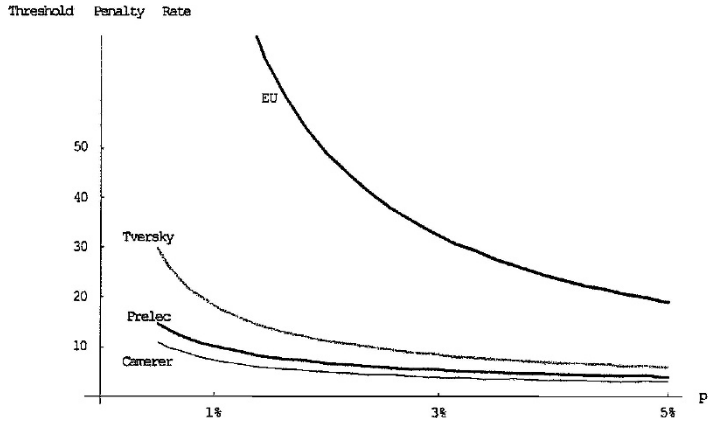

Figure 3. Threshold penalty rates (θ^*_{EU} and θ^*_{RDEU}) needed to ensure full compliance, as function of the audit probability p.

θ belongs to the interval $[\theta^*_{RDEU}, \theta^*_{EU})$, RDEU predicts full compliance, as opposed to the EU model under which some cheating will obtain. Generalizing the Allingham-Sandmo-Yitzhaki model to RDEU therefore strengthens the deterrence effect of the penalty rate.

These results are illustrated graphically in figures 2 and 3. Figure 2 presents the critical audit probabilities that ensure full compliance (on the vertical axis) for penalty rates θ that vary between 0 and 3, for two specifications of the probability weighting function.[11] Figure 3 reports the penalty rate needed to ensure full compliance, as a function of the audit probability. Note that the results presented in figures 2 and 3 are independent of $u(.)$. For penalty rates that are greater than 0.5, the Prelec specification predicts threshold audit probabilities that are lower than those predicted by Tversky and Fox, which in turn are lower than those predicted by EU. This ordering is inverted for penalty rates below 0.5 (which corresponds, given our parameterization, to Condition 1(ii) being violated), though the ensuing critical audit probabilities are much too high. For threshold penalty rates, Camerer and Ho predicts the lowest rate, followed by Prelec, and finally Tversky and Fox. In contrast, the threshold audit probabilities predicted by the EU model are so high (above 20 even for an audit probability of 0.05) as to be unrealistic by any standard. In terms of threshold audit probabilities and penalty rates, the Camerer and Ho specification of the probability weighting function would therefore appear to offer the best chance of providing an explanation for the tax compliance puzzle.

3.2. Compliance rates

Proposition 2 established that compliance rates under RDEU should be greater than in the EU case. How large are the quantitative differences between the two models? Experimentation revealed that the specification of $u(.)$ was the crucial element in determining the simulated extent of compliance. In the four panels of figure 4 we present simulations of compliance rates ($\frac{z-y}{y}$) for various specifications of $u(.)$, as a function of the audit probability (which varies between 0.005 and 0.05, roughly the orders of magnitude that one observes in US data over the last 50 years), and with $t = 0.30$ (again, this corresponds to a reasonable

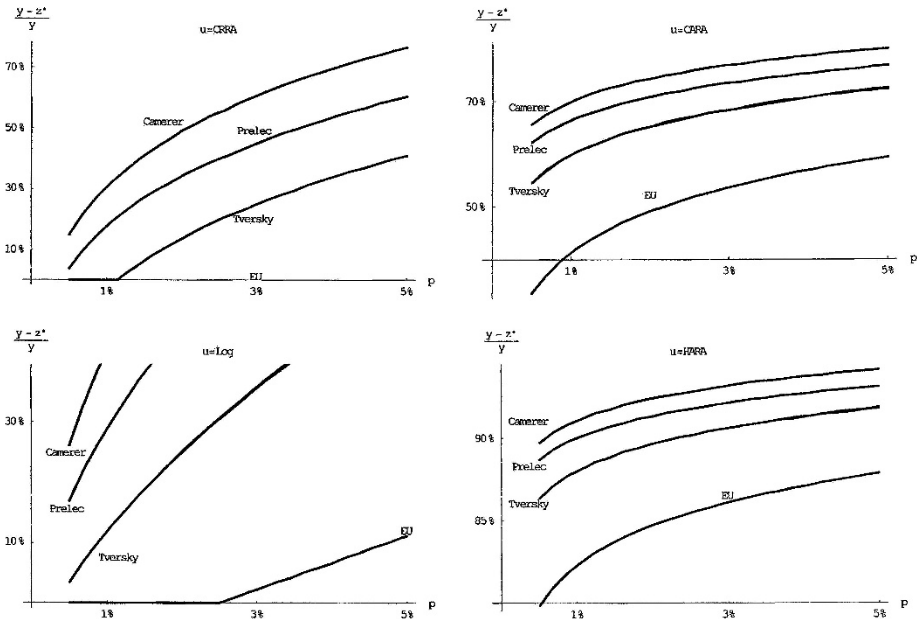

Figure 4. Simulated compliance rate $(\frac{z^*-y}{y})$ for four specifications: CRRA: $u(x) = \frac{x^{-0.8}}{-0.8}, \theta = 2$; CRRA: $u(x) = \ln x, \theta = 3$; CARA: $u(x) = -\frac{e^{-20x}}{20}, \theta = 0.5$; HARA: $u(x) = \frac{1-k}{A(2-k)}(Ax + B)^{\frac{2-k}{1-k}}, A = 2, B = 0, k = 1.03, \theta = 0.5$.

level in light of the US experience). While many more specifications were tried, the results presented here are representative of our findings. Four aspects of our results are worth underlining.

First, for CRRA and CARA utility functions, compliance rates are much too low when compared to observed levels, at least in the US, while a HARA specification for the utility function yielded orders of magnitude that are reasonably close to reality (more on this below). Second, the ranking of the RDEU simulations in terms of compliance rates is always the same, with the Camerer and Ho parameterization of the probability weighting function yielding the highest compliance rates, followed by Prelec, and finally Tversky and Fox. Third, the EU specification always yields compliance rates that are *much* lower than its RDEU counterparts. This is particularly true in the case of the CRRA specifications, where the EU specification yields *zero* compliance rates for audit probabilities below 5% when $-\frac{u''(c)}{u'(c)}c = 1.8$, with the corresponding figure being approximately 2.5% when the utility function is logarithmic. Fourth, even with $-\frac{u''(c)}{u'(c)} = 20$, the CARA specification is unable to yield realistic levels of compliance. The upshot is that only a HARA specification for $u(.)$ appears to be capable of generating compliance behavior, for realistic parameter values, that squares with available evidence.

3.3. *Confronting "real" data*

How do the two specifications –EU and RDEU– compare in terms of their ability to predict true compliance rates over the long term? In figure 5, we present simulation results for the underreporting rate (z/y) for the US over the past half-century (1947–2000), which we

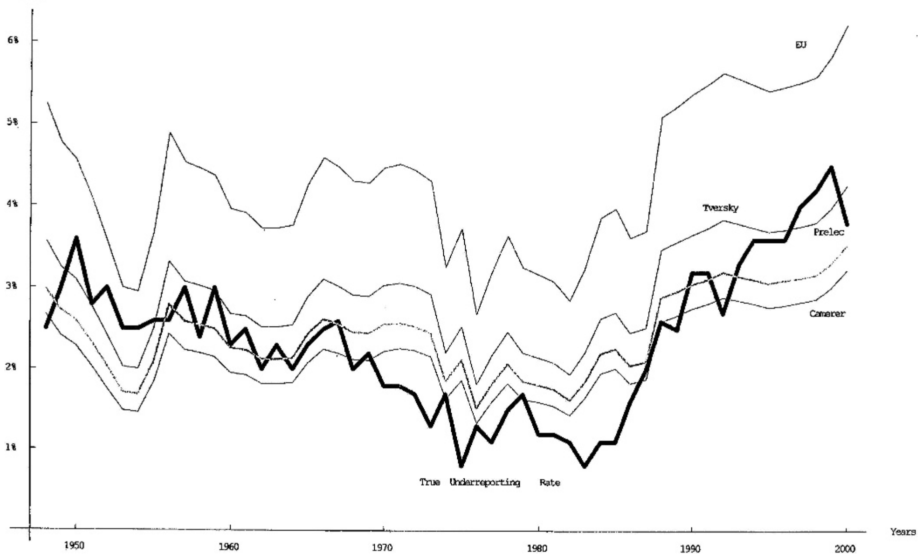

Figure 5. Actual versus simulated underreporting rate ($\frac{z^*_{EU}}{y}$, $\frac{z^*_{RDEU}}{y}$) for the US, 1947–2000.

compare to the "true" underreporting rate.[12] The empirical counterpart to z is given here by the US Bureau of Economic Analysis (BEA) "AGI [Adjusted Gross Income] wage gap for wage and salary income", which represents the difference between the BEA's estimate of wages and salaries and taxpayer-reported wages and salaries.[13] This measure is then adjusted to account for "legitimate non-filers" (mainly low-income individuals who are not required to file a tax return) using evidence from the 1988 TCMP study. The tax rate (t) is a weighted average marginal tax rate on ordinary income (excluding social security and medicare).[14] Income (y) is given by the Census Bureau's Current Population Survey estimate of median wage and salary income.[15] The audit probability p is given by the "face to face" audit rate, as published by the IRS.[16] The penalty rate is assumed to be constant and equal to 0.5. This is not an unreasonable figure for the US, where the *monetary* penalty rate varies between 0.25, and 0.75.[17] This does not, however, take potential incarceration into account (see e.g. Andreoni, Erard, and Feinstein [1998]), or the fact that audits of prior year tax returns, when performed in conjunction with an audit of the current year tax return, can be thought of as an extended penalty.

The utility function used in the simulation is of the HARA class: simulations using the CARA and CRRA specifications were far less successful and failed to generate the substantial fall in tax-evasion that occurred between the mid-seventies and mid-eighties. The simulation results presented in the previous subsection, in which the HARA form for $u(.)$ was demonstrably superior in terms of the simulated level of compliance, also led us to prefer this specification. Three remarks are in order concerning this simulation.

First, it is clear that both the EU and RDEU models (irrespective of the parameterization chosen for the probability weighting function) do a surprisingly good job of tracking changes in actual underreporting rates. In quantitative terms, the correlation between the actual underreporting rate and each of the simulated series is approximately equal to 0.76. If one runs a simple regression of the actual underreporting rate on any of the simulated series, the resulting R^2 is equal to 0.57, which is surprisingly high for such a simple model.

Second, the most glaring difference between the simulated RDEU and EU series obtains in terms of their means. While the mean "true" underreporting rate is equal to 2.37%, EU predicts a mean underreporting rate of 4.31%, with the corresponding figures for the RDEU simulations ranging from 2.16% (Camerer and Ho) to 2.93% (Tversky). As expected, EU predicts a much higher rate of underreporting than does RDEU: the mean difference between the actual and simulated EU underreporting rates is equal to 1.93% and is highly significant (t-statistic = 22.53), while the corresponding difference is equal to 0.07% and is statistically insignificant (t-statistic = 0.89) for the Prelec specification. RDEU therefore predicts mean underreporting rates that are in line with US historical evidence, while the corresponding EU model predicts underreporting rates that are almost twice as high as they should be.

Third, while RDEU is more successful than EU in predicting mean underreporting rates, it is less so in terms of the amplitude of their variations over time (and this despite the identical correlation coefficients mentioned above). The standard deviation of the true underreporting rate is equal to 0.0091: EU predicts roughly the same number (0.0090), while all RDEU specifications are off the mark by a factor of almost one half (standard deviation of 0.0050 for Prelec). RDEU therefore tends to underestimate the variation in underreporting rates caused by changes in the underlying variables (t, y and p).[18]

4. Concluding remarks

Considering the Allingham-Sandmo-Yitzhaki problem under RDEU axiomatics has allowed us to bridge at least part of the gap between observed levels of compliance and theoretical predictions. Intuitively, RDEU axiomatics allow one to do this by introducing "pessimism" into the individual's decision-making process in that the taxpayer will overestimate the probability of audit. Contrary to Bernasconi [1998], we believe that social or ethical factors may still account for a portion of the tax compliance puzzle, insofar as they affect the probability weighting function, although we have not developed this point here. Experimental evidence would be extremely useful in this context, and should be extended to countries outside of the US.[19] An example includes the concept of "competence" as defined by Heath and Tversky [1991], which could explain the use of accountants for establishing tax returns. Finally, institutional factors, such as third party reporting, may be the key to understanding tax compliance, although our focus would be on how these elements affect the probability perception function.

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Notes

1. As applied to tax compliance, we prefer the RDEU approach to its main competitor, Cumulative Prospect Theory (CPT), developed in a series of papers beginning with Kahneman and Tversky [1979]. CPT does, as noted by Cowell [2003], present a number of advantages. First, individuals "edit" the information associated with the underlying lotteries. Second, and contrary to the RDEU approach, CPT allows one to distinguish the value function (as opposed to the utility function) from the weighting function associated with the probabilities of given gains or losses. On the other hand, and in the context of tax compliance we believe this argument to be the clincher, CPT imposes a reference point. The choice of this reference point, or *status quo*, is crucial for any ensuing results. In the case of tax compliance, this would involve choosing arbitrarily between after- or pre-tax income as the reference point. Moreover, the existence of a reference point implies that consumers should be indifferent to wealth effects, which runs counter to most empirical evidence that indicates that the degree of fraud is correlated with the consumer's level of income. Note that Cox [1984] finds a U-shaped relationship between income and the rate of underreporting, which is inconsistent with all currently-used theoretical models, while Bloomquist [2003a] presents US evidence of a higher rate of underreporting for lower income brackets. Current received wisdom among practitioners is that the relationship between income and the rate of underreporting can at best be described as non-linear.
2. Note that we assume $z \geq 0$. If one had $z < 0$, the structure of the optimization program implies that the taxpayer would receive a reward for over-declaration (this follows because of the formulation in terms of a penalty: $-\theta t z^* > 0$ if $z^* < 0$). As such, we prefer to assume, as in Andreoni, Erard, and Feinstein [1998] that overdeclaration is irrational.

3. We present the definition of $RDEU[P(z)]$ in terms of a decumulative density function (i.e., 1 *minus* the cumulative density), whereas, in his illustration of the kink, Bernasconi [1998] uses a cumulative density function formulation.
4. The second order condition holds because we shall assume strict concavity of $u(.)$. The FOC is therefore sufficient as well as necessary.
5. In what follows, subscripts will denote partial derivatives, i.e. $\frac{\partial f}{\partial x} = f_x$.
6. Condition 1 could be replaced by the much stronger requirement that $\varphi(1-p) \leq 1-p, \forall\, p$, which would correspond to a convex probability transformation function, but this would preclude us from using the inverse S-shaped functions that have been found in experimental work.
7. Note that one can also establish, for $q < \underline{p}^*_{RDEU}(\theta)$, and under the assumption that $-\frac{u''(.)}{u'(.)}$ is decreasing, that $0 < \frac{dz^*_{RDEU}}{dy} < \frac{dz^*_{EU}}{dy}$ and $\frac{dz^*_{RDEU}}{dt} < \frac{dz^*_{EU}}{dt} < 0$. The proof is available upon request.
8. That is, $\varphi(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{\frac{1}{\gamma}}}$, in which we use Abdellaoui's [2000] specification, i.e. $\gamma = 0.7$, with a CRRA utility function ($u(x) = \frac{x^{1-\sigma}}{1-\sigma}$, $\sigma = 1.8$), and $\theta = 2$, $t = 0.30$.
9. While some of these probability weighting functions were developed in a CPT context, they are readily transposed to the RDEU approach (see e.g. Camerer [1994]). Our simulations, carried out in Mathematica 5.0, are available upon request.
10. Some initial experimental evidence can be found in Alm, Jackson, and McKee [2004].
11. We were unable to determine, even numerically, the threshold audit probability for the Camerer and Ho (1994) probability weighting function.
12. We are extremely grateful to Kim Bloomquist of the IRS for making these data available to us. See Bloomquist [2003b] for further details.
13. Available online at: http://www.bea.gov/bea/ARTICLES/2004/04April/0404PI&AG.pdf.
14. The raw data concerning the amount of income reported by taxpayers in different tax rate brackets are available online at: http://www.irs.gov/taxstats/article/0,,id=96586,00.html. Bloomquist [2003b] then uses Census Bureau data defining the upper and lower income bounds for population quintiles to derive the average marginal tax rates.
15. Available online at: http://www.census.gov/hhes/income/histinc/p53.html.
16. Available online at: http://trac.syr.edu/tracirs/trends/current/audpctcompare_ind.html and http://www.irs. gov/taxstats/article/0,,id=102174,00.html.
17. Note that Bernasconi (1998) sets $\theta = 3$ in his simulations.
18. It is interesting that this last finding is perfectly in line with the theoretical results mentioned in note 7, where it was noted that the comparative statics of the RDEU model are "weaker", with respect to changes in y and t, than for the EU model.
19. See Cummings, Martinez-Vasquez, and McKee [2001], for some experimental evidence for South Africa, Botswana and the US.

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